

## 5.1 Determinants

### 513. Second Order Determinant

$$\det A = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

### 514. Third Order Determinant

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$

### 515. Sarrus Rule (Arrow Rule)

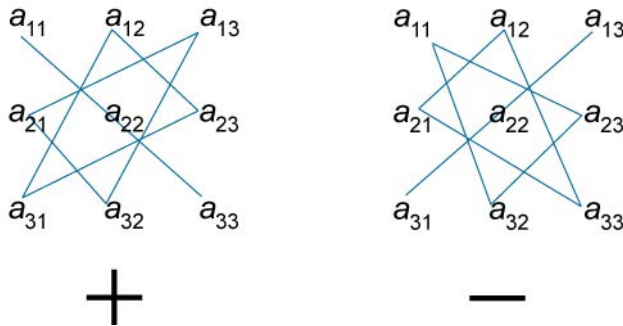


Figure 72.

### 516. N-th Order Determinant

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{i1} & a_{i2} & \dots & a_{ij} & \dots & a_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nj} & \dots & a_{nn} \end{vmatrix}$$



**517. Minor**

The minor  $M_{ij}$  associated with the element  $a_{ij}$  of  $n$ -th order matrix  $A$  is the  $(n-1)$ -th order determinant derived from the matrix  $A$  by deletion of its  $i$ -th row and  $j$ -th column.

**518. Cofactor**

$$C_{ij} = (-1)^{i+j} M_{ij}$$

**519. Laplace Expansion of  $n$ -th Order Determinant**

Laplace expansion by elements of the  $i$ -th row

$$\det A = \sum_{j=1}^n a_{ij} C_{ij}, \quad i = 1, 2, \dots, n.$$

Laplace expansion by elements of the  $j$ -th column

$$\det A = \sum_{i=1}^n a_{ij} C_{ij}, \quad j = 1, 2, \dots, n.$$

